ANALYSIS OF WAVEGUIDE JUNCTIONS BY RANK REDUCTION

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ABSTRACT

A new low-rank spectral expansion technique for solving intractable equations obtained from waveguide equivalence theorem decompositions is illustrated for a hybrid junction. The technique takes into account higher-order modes and eliminates spurious resonances.

Analysis of Waveguide Junctions by Rank Reduction

The integral equations and corresponding matrix equations that represent scattering at a waveguide discontinuity are often ill-conditioned. Inversion of such matrices is inaccurate for even large-order truncations, but advantage may be taken of the often relatively low effective rank of the matrix to ease the inversion and eliminate spurious resonances that arise from severe truncation.

The technique is illustrated for scattering in the waveguide hybrid junction shown in Figure 1. Previous solutions have required the sheet to be thin enough to make the waveguide misalignment negligible, and the frequency to be high enough for quasi-optical analysis to be applicable[1-3], or else have required direct inversion of large-order, ill-conditioned matrices[4]. Analysis of the hybrid is important in band diplexer design, to achieve satisfactory frequency band separation and to identify spurious modes that may degrade system performance[5-7].

The hybrid with a single mode incident from port 1 is equivalent to the superposition of four symmetrically excited 4-port structures in which planes S_1 and S_2 are replaced by electric or magnetic walls. Thus only the 1-port structure shown at the left in Figure 2 needs to be analyzed. Field equivalence theorems[8-15], applied as in the figure, reduce the complicated geometry to two uniform waveguides excited by unknown electric and magnetic current sheets, J, M. The simplifications are valid because the media and boundaries of regions devoid of fields can be freely altered without affecting the fields in the other regions.

The currents can be determined by requiring their radiated fields to annihilate each other throughout the null-field regions[16,17]. Substitution of expansions for the currents into the integral equations that express the scattered fields in terms of the unknown currents yields the exact matrix for the unknown current coefficient vector c in terms of the known incident modal amplitude vector s,

$$Gc = s \; . \tag{1}$$

The elements of G are formed from integrals involving scalar products of normal modes of the uniform waveguides with the chosen current expansion functions.

Partition (1) as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix} \tag{2}$$

Square matrix A is what G is truncated to if only the first few current expansion terms are kept and only the first few modes are required to satisfy the null-field condition. The first few current expansion coefficients are contained in x; the higher-order ones are in y. The single incident modal amplitude is in the subvector r of the otherwise null source vector s.

A crude solution to (2) is

$$x' = A^{-1} r \; . \tag{3}$$

An exact solution is

$$\begin{aligned} x &= (A - BD^{-1}C)^{-1} r \\ x &= -\, D^{-1} C x \; . \end{aligned} \tag{4}$$

Since D is ill-conditioned, D^{-1} cannot be readily evaluated, especially when D is of high order. Solutions more accurate than (3) can nevertheless be obtained by substituting for D^{-1} an approximation D^{-}, the group inverse of D[18]. A useful solution, more accurate than (3) in that high-order terms are not neglected, is

$$\begin{aligned} x &= (A - BD^{-}C)^{-1} r \\ y &= -\, D^{-} C x \; . \end{aligned} \tag{5}$$

The group inverse $D^{-} = f(gf)^{-2}g$ is obtained from the factorization $D = fg$ that results from retaining the K dominant singular values in the singular-value decomposition of D[18-21]. Matrices f and g are N×K and K×N respectively, when D is, for practical purposes, truncated to a large but finite size N×N, with N usually much larger than K. The largest matrix that needs to be inverted is gf, which is only K×K and well-conditioned.

Numerical results[22] are presented for the zero thickness unity dielectric-constant hybrid, or empty waveguide cross junction. Figure 3 shows the scattering coefficient from port 1 to port 3 obtained from the severely-truncated matrix (G=A is 4×4); it also presents the effects of higher-order terms obtained by rank reduction (D is 8×8 and approximately rank 2), and compares these with the exact results of Bouwkamp[23]. There is agreement with the exact results to within a fraction of a dB.

Near isolated frequencies, resonances with no physical basis appear in the solution that uses the severely-truncated matrix. A typical such resonance is shown in Figure 4 on an expanded frequency scale. At the spurious resonance frequency the G matrix becomes nearly singular, one eigenvalue being at least two orders of magnitude smaller than the other three. By applying rank reduction, the spurious resonance is removed, as shown in Figure 4 for a rank-3 approximation.

Typically, scattered fields obtained from a rank-reduced 4×4 inversion of A or D agree with those from a direct inversion of a 12×12 G-matrix to within a few percent.

Typical electric and magnetic current sheets on surface S_a appear in Figure 5. Only two terms are needed in the expansions for the currents. Observe that the shapes are characteristic of the dominant mode field and consistent with the boundary conditions at the walls.

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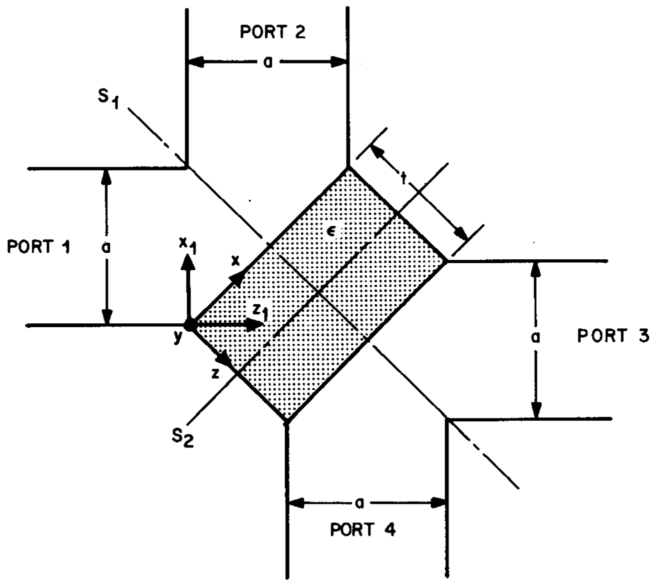

Fig. 1-Top view of hybrid junction formed by two crossed waveguides of width a whose junction is traversed at a 45-degree angle by a dielectric sheet of thickness t and relative dielectric constant ε.

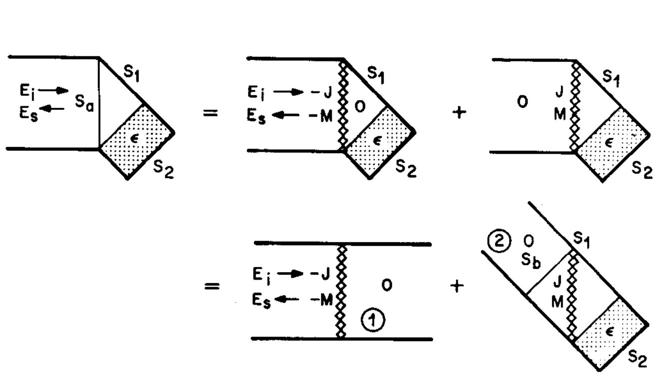

Fig. 2-Equivalence-theorem decomposition into two waveguides, each having a null-field region (0). The null-field media are then altered to yield two simple waveguide structures.

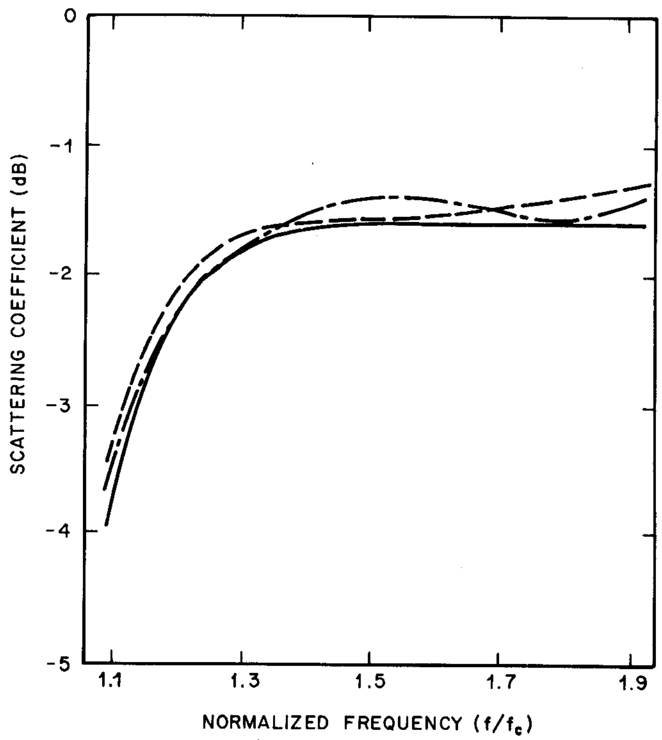

Fig. 3-Transmission coefficient from port 1 to port 3 for the empty junction. The solid line is for the severely-truncated matrix method, the broken line shows the effect of higher-order terms using rank-reduction, and the dashed line is the exact result given by Bouwkamp[23]. TE_{01} cutoff is $f/f_c = 1$.

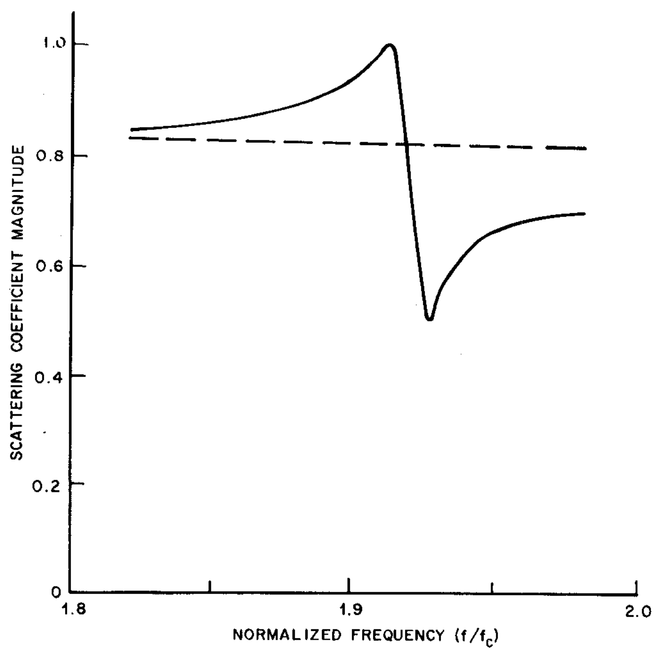

Fig. 4-Elimination of spurious resonances in transmission coefficient from port 1 to port 3 for the empty junction. Solid line is from direct inversion of G (4×4). Dashed line is from rank-3 approximate inversion. TE_{01} cutoff is $f/f_c = 1$.

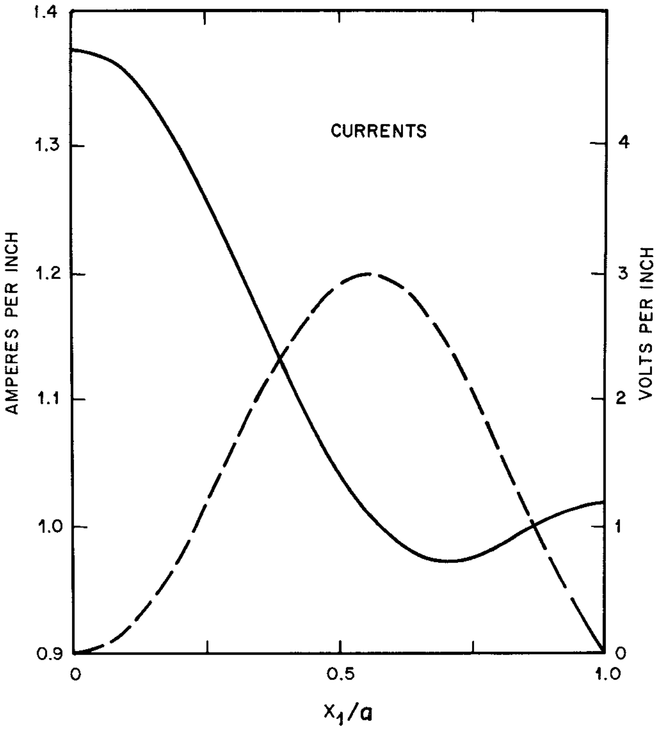

Fig. 5-Electric (solid) and magnetic (dashed) current sheet magnitudes vs. distance along discontinuity surface S_a for the 1-port of Figure 2, with S_1 and S_2 electric and magnetic walls, respectively. Frequency is 1.85 times TE_{01} cutoff frequency.